Fatigue and Fracture of Materials

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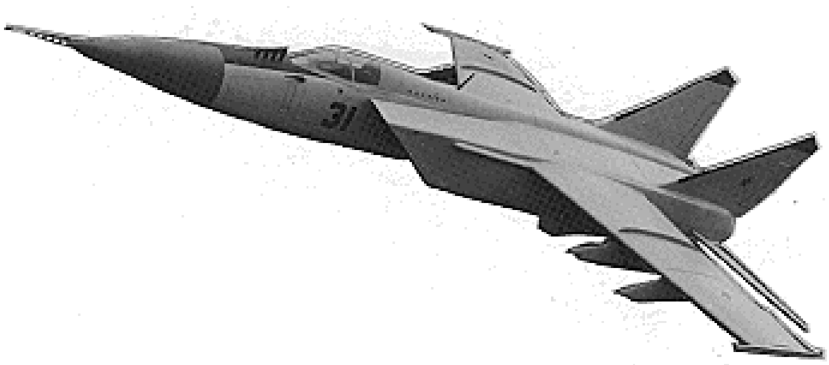
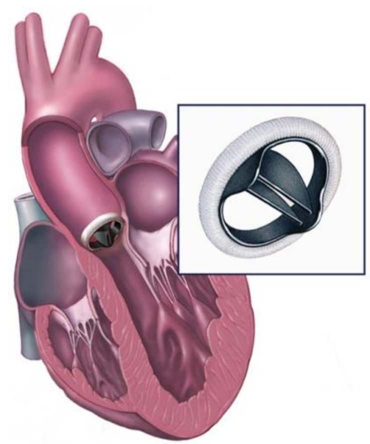
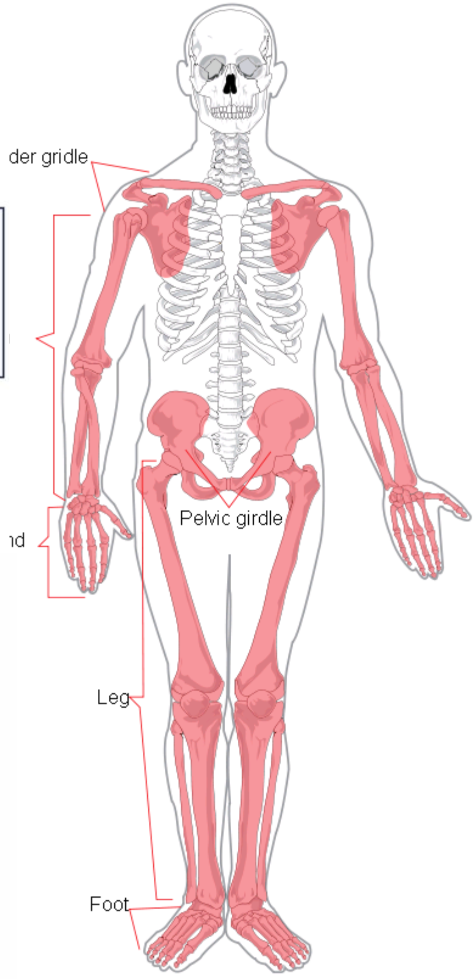
Fatigue and Fracture







Objectives of Course

- This course presents a graduate level introduction to fatigue and fracture of materials
- The mechanisms and mechanics and fatigue and fracture are introduced
- These are integrated with basic concepts in finite element modeling and simulations
- The frontiers of research are also presented at the end of the course

Course Outline

- Tuesday 6 September
  - Introduction to Mechanical Properties
  - Introduction to Elasticity
  - Introduction to the Finite Element Method
  - Abaqus Software Installation and Configuration

- Wednesday 7 September
  - Introduction to Plasticity
  - Introduction to Notches and Fracture Mechanics
  - Review of Truss Problems
  - Finite Element Modeling of Truss Problems

Course Outline

- <u>Thursday 8 September</u>
  - Fracture and Toughening of Materials
  - Fundamentals of Fracture Mechanics
  - Finite Element Modeling of Plate With Hole

- <u>Friday 9 September</u>
  - Fundamentals of Fracture Mechanics – cont
  - Introduction to Fatigue
  - Finite Element Modeling of Crack Problems

Course Outline

- <u>Saturday 10 September</u>
  - Case Study of Contact
  - Case Study of Adhesion
  - Finite Element Modeling of Hertzian Contact and Research Discussions

Approach to the Course

- Intensive combination of lectures and hands on computer-based modules
- Lecture
- Break
- Lecture
- Lunch
- Finite element modeling lecture
- Finite element modeling

Definitions of Stress and Strain

- The mechanical properties of materials describe their characteristic responses to applied loads and displacements
- However, most texts relate mechanical response/behavior to stress and strain
- We will, therefore, begin with some basic definitions of stress and strain
- We will begin with simple definitions and proceed to more rigorous definitions

Basic Definitions of Stress

- The forces applied to a body may be resolved into components that are perpendicular or parallel to the surface of the body
- Axial stress = $\sigma = \frac{\text{Applied load (Normal to Surface)}}{\text{Cross-Sectional area}} = \frac{P_n}{A}$
- Shear stresses = $\tau = \frac{\text{Applied load (Parallel to Surface)}}{\text{Cross-Sectional area}} = \frac{P_s}{A}$

<u>Simple Tension</u> <u>Simple Compression</u> <u>Pure Shear</u>

More Rigorous Definitions of Stress

- More rigorous definitions of stress are needed when the cross-sectional areas are not uniform
- Uniaxial axial and shear stresses defined as the limits of the following expressions as $dA \rightarrow 0$

$$\sigma = \lim_{dA \to 0} \left(\frac{P_n}{dA} \right)$$

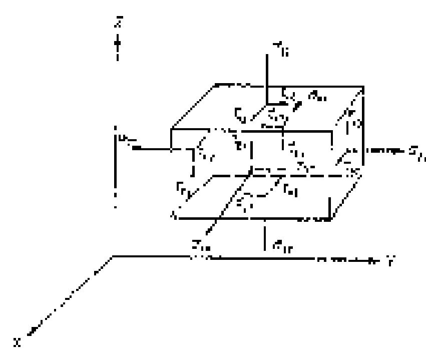
and

$$\tau = \lim_{dA \to 0} \left(\frac{P_s}{dA} \right)$$

Stress as a Tensor Quantity

- Stress is not a vector quantity that can be defined by magnitude and direction of the force
- Definition must also specify area normal to the element
- Stress is a second order tensor that requires the specification of two direction normals

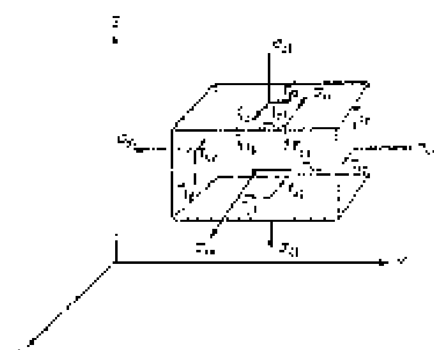
Stress on an Element



State of Stress and Sign Convention

- There are nine stress components on the faces of the cube
- Special sign convention is used to define the directions of stresses that act on an element
  - first suffix, i, in σ_{ij} or τ_{ij} terms corresponds to the direction of the normal to the plane
  - second suffix, j, corresponds to the direction of the load

<u>State of Stress on an Element</u>



Stress Tensor for a Generalized Three-Dimensional Stress State

- In the Cartesian coordinate system – the complete stress tensor for a generalized 3-D stress state is given by

$$[\sigma]=\begin{bmatrix}\sigma_{xx} & \tau_{xy} & \tau_{xz}\\ \tau_{yx} & \sigma_{yy} & \tau_{yz}\\ \tau_{zx} & \tau_{zy} & \sigma_{zz}\end{bmatrix}$$

- In the cylindrical and spherical coordinate systems

Cylindrical Coordinates

$$[\sigma]=\begin{bmatrix}\sigma_{rr} & \tau_{r\theta} & \tau_{rL}\\ \tau_{\theta r} & \sigma_{\theta\theta} & \tau_{\theta L}\\ \tau_{Lr} & \tau_{L\theta} & \sigma_{LL}\end{bmatrix}$$

Spherical Coordinates

$$[\sigma]=\begin{bmatrix}\sigma_{rr} & \tau_{r\theta} & \tau_{r\phi}\\ \tau_{\theta r} & \sigma_{\theta\theta} & \tau_{\theta\phi}\\ \tau_{\phi r} & \tau_{\phi\theta} & \sigma_{\phi\phi}\end{bmatrix}$$

Choice of Coordinate System and Some General Observations on Stress on an Element

- This depends on the geometry of the solid that is being analyzed

- Example – will choose cylindrical coordinate system in the analysis of a cylinder or body with cylindrical symmetry

- Note that there are only 6 independent stress components in any of the coordinate systems

- three axial components
- three shear components

Basic Definitions of Strain

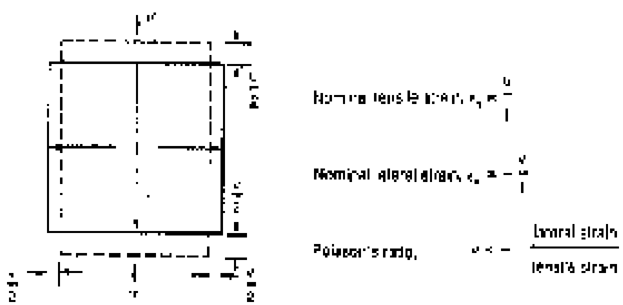
- Applied loads or displacements result in changes in the shape of a solid

- Axial and shear strains can be defined as

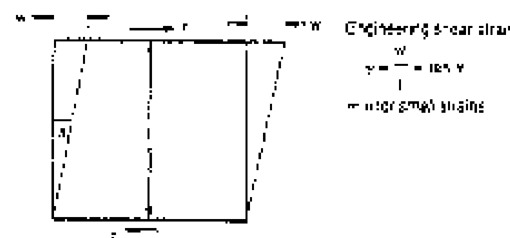
$$\varepsilon = u/l_o$$

$$\gamma = w/l = \tan\theta$$

Axial Strain



Shear Strain



More Rigorous Definitions of Strain

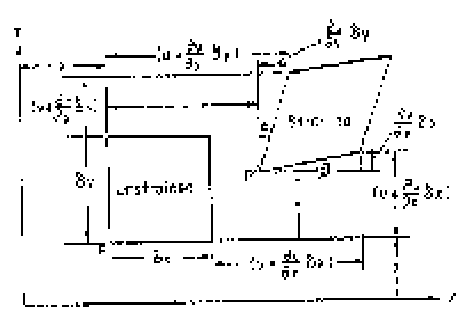
- Earlier definitions assume that stresses are uniform across area elements
- We may also define three axial strains (ε_{xx}, ε_{yy}, ε_{zz}) and three shear strains (γ_{xy}, γ_{yz}, γ_{zx}) in 3-D state of strain

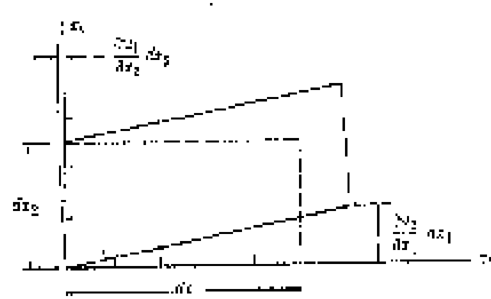
$$\varepsilon_{xx} = \frac{\left[u + \left(\frac{\partial u}{\partial x}\right)dx\right] - u}{dx} = \frac{\partial u}{\partial x}$$

$$\varepsilon_{xy} = \frac{\left[v + \left(\frac{\partial v}{\partial x}\right)dx\right] - v}{dx} = \frac{\partial v}{\partial x}$$

Definitions of Strains

Definitions of Rotations



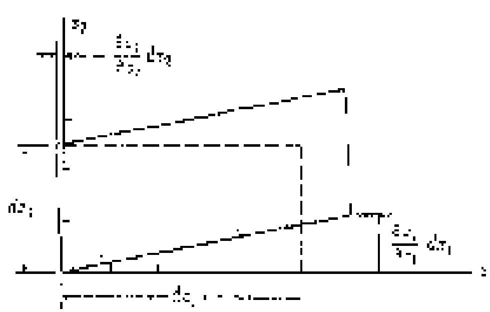


Accounting for Rotation Terms

- For stress-induced deformation – we subtract out the rotation terms to describe local changes in shape
- Average rotation about x-y plane is given by

$$\omega_{xy} = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

<u>Rotation About z-Axis</u>



Basic Expressions for Rotation Components

- May obtain ω_{yz} and ω_{zx} by cyclic permutations of the (x,y,z) terms and corresponding u,v,w displacement terms

$$\omega_{yz} = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) \qquad \omega_{zx} = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)$$

- The components of the rotation matrix are given by

$$[\omega_{ij}] = \begin{bmatrix} 0 & \omega_{xy} & \omega_{xz} \\ \omega_{yx} & 0 & \omega_{yz} \\ \omega_{zx} & \omega_{zy} & 0 \end{bmatrix} = \begin{bmatrix} 0 & \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} & \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \\ \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} & 0 & \frac{\partial v}{\partial z} - \frac{\partial w}{\partial y} \\ \frac{\partial w}{\partial x} - \frac{\partial u}{\partial z} & \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} & 0 \end{bmatrix}$$

Strain Matrix for Stress-Induced Deformation

- Subtracting the rotation matrix from the original strain matrix now gives

$$[\varepsilon_{ij}] = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix} = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) & \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \\ \frac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) & \frac{\partial v}{\partial y} & \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) \\ \frac{1}{2}\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right) & \frac{1}{2}\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right) & \frac{\partial w}{\partial z} \end{bmatrix}$$

- Sign convention is similar to that for stress
  - first term in ε_{ij} corresponds to the direction of the normal to the plane
  - second term corresponds to the direction of the displacement induced by the applied strain

Tangential Strain Components in Engineering Problems

- The factor ½ often not included in several engineering problems where only a few strain components are applied

- The tensorial strains (ϵ_{ij} terms) are then replaced by tangential shear strain terms (γ_{ij} terms) that are given by

$$\gamma_{ij} = 2\epsilon_{ij}$$

- The strain matrix for stress-induced deformations is then given by

$$\left[\gamma_{ij}\right] = \begin{bmatrix} 0 & \gamma_{xy} & \gamma_{xz} \\ \gamma_{yx} & 0 & \gamma_{yz} \\ \gamma_{zx} & \gamma_{zy} & 0 \end{bmatrix} = \begin{bmatrix} 0 & \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} & \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \\ \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} & 0 & \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \\ \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} & \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} & 0 \end{bmatrix}$$

- Above shear strain components often important in plastic flow problems

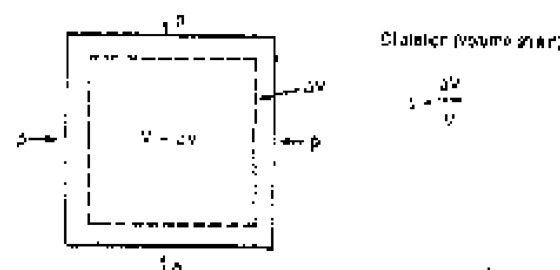
The Importance of the Different Strain Components

- Tangential strain components often important in problems involving plastic flow

- Axial strain components and volumetric strain important in brittle fracture (bond rupture) problems

$$\varepsilon_V = \frac{\Delta V}{V} = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}$$

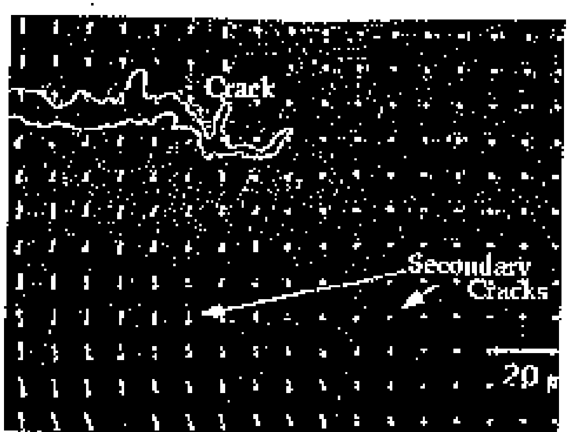
<u>Basic Definition of Volumetric Strain</u>

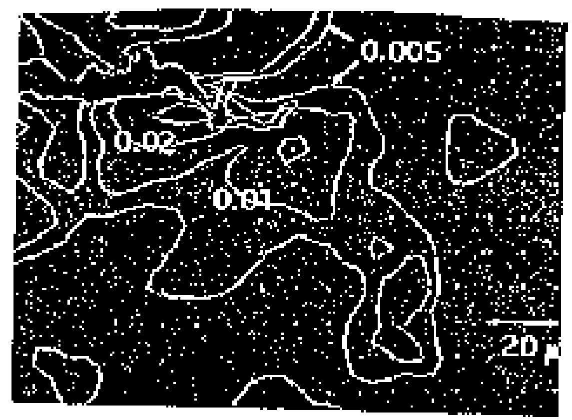


Measurement of Local Strains from Grid Displacements

- Local strains and rotations can be measured from grid displacements
- Basic ideas are quite simple – simply measure displacements and use equations to find strain/rotation component
- Resolution in displacement measurements is the key

Schematic of Grid Displacement Method





Limitations of Small Strain Formulations

- Above definitions apply to cases where the displacements are small
- Large scale formulations are needed when displacements are large e.g. Lagrangian strain formulations

$$\eta_{xx} = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2\right)$$

$$\eta_{yy} = \frac{\partial v}{\partial y} - \frac{1}{2}\left(\left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2\right)$$

$$\eta_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + \frac{1}{2}\left(\frac{\partial u}{\partial x}\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\frac{\partial v}{\partial y}\right)$$

Summary and Concluding Remarks

- Basic introduction to stress and strain presented along with their limitations/applications
- Simple definitions were followed by more rigorous descriptions
- Components of the stress and strain tensors were presented – (3 axial and 3 shear terms)
- Deformation-induced strains terms were obtained by subtracting rotations from overall strains
- Tangential strain terms were defined – important in engineering problems involving plastic flow
- Lagrangian strain formulation presented for larger displacements